Int. J. Heat Mass Transfer. Vol. 6, pp. 915–923. Pergamon Press 1963. Printed in Great Britain.

THE THERMODYNAMIC COUPLING BETWEEN HEAT AND MASS TRANSFER IN FREE CONVECTION WITH HELIUM INJECTION*

O. E. TEWFIK† and JI-WU YANG‡

(*Received* 27 *December* 1962)

Abstract—Measurements are described of local heat transfer by free convection under steady state conditions around a 2-in o.d. circular cylinder mounted with its axis horizontal in an isothermal large enclosure, at several levels of wall temperature ranging from about 20°F below ambient, to about 80°F above ambient.

Helium was injected through the porous wall of the cylinder into the boundary layer at a uniform rate per unit area of outside cylinder surface, in the range 1·00–7·50 $lb_m/h\ ft^2$.

Analysis of the results showed that when the convective heat flux is zero, the wall temperature, in this case termed the adiabatic wall temperature, was higher than ambient temperature by up to 190°F, depending upon the location around the cylinder, and the helium injection rate. This phenomenon, so far unreported in free convection, is similar to the findings in references [1, 2], and hence is due to the diffusion thermo-effect.

The local heat-transfer coefficient was defined in terms of the adiabatic wall temperature, and determined to be essentially independent of the temperature difference between wall and ambient. At the lower stagnation point, its value increased rapidly with injection, but then levelled off to an almost constant value when the injection rate exceeded about 2 $lb_m/h\ ft^2$. At a given injection rate, the heat-transfer coefficient decreased from a maximum value at the lower stagnation point to a minimum value at the upper stagnation point.

At a given location on the cylinder circumference, the adiabatic wall temperature increased steadily as the injection rate increased. At a given injection rate, the adiabatic wall temperature increased from a minimum value at the lower stagnation point to a maximum value at the upper stagnation point.

The body forces at work in the binary boundary layer are discussed in their importance on heat transfer.

NOMENCLATURE

- b, thickness of stainless steel porous shell 0·032 in;
- C_p, specific heat of helium at constant pressure;
- d, outside diameter of cylinder = 2 in;
- h, coefficient of heat transfer by free convection, equation (4);
- k, thermal conductivity;
- k_s, thermal conductivity of stainless steel shell;
- $\dot{m}$, mass injection rate of helium per unit area of outside cylinder surface;
- q_w, heat flux by free convection, equation (1);
- q_r, net heat flux, by radiation, equation (2);
- q_c, heat flux by circumferential conduction, equation (3);
- r, outside radius of cylinder = 1 in;
- T, temperature;
- T_a, adiabatic wall temperature at which $q_w = 0$;
- T_c, temperature of helium inside cylinder;
- T_s, temperature of the inside surface of the enclosure;
- θ, angle measured from radius through lower stagnation point, Fig. 2.

* Communication from the Heat Transfer Laboratory, University of Minnesota, Minneapolis 14, Minn.

This research was mainly supported by the U.S. Air Force through the AFOSR of the ARDC under contract number AF 49(638)-558.

† Currently Project Engineer, Corning Glass Works, Corning, N.Y.

‡ Research Assistant, Department of Mechanical Engineering, University of Minnesota.

Subscripts

w, cylinder outside wall;
∞, ambient, inside the enclosure and far away from cylinder wall.

1. INTRODUCTION

SIMULTANEOUS transfer of mass and heat from a surface take place in many engineering applications, as for example in the ablation cooling of missiles and re-entering space vehicles. Recent measurements of forced convection heat transfer with helium injection into a boundary layer [1, 2], showed a strong coupling between the transport of heat and the transport of helium, or diffusion thermo-effects in the boundary layer. However, no experimental or theoretical analyses to investigate such effects in free convection exist to date. Hence the following experimental investigation was undertaken at the Heat Transfer Laboratory to initiate research in this direction.

2. APPARATUS AND METHOD

The model was in the form of a 2-in o.d. circular cylinder, with a 0·032-in thick porous shell woven from stainless steel wire. The shell was lined with fiber glass filter paper to a thickness of 0·3 in approximately. Fig. 1 shows details of the model construction and instrumentation.

The model was mounted with its axis horizontal in an approximately isothermal large enclosure in the form of a double walled steel cylinder, 2 ft i.d., and 4 ft high as shown in Fig. 2. Helium from a bank of twenty bottles was heated or cooled to the desired temperature level by passing it through an electric heater and/or a cooler. It was then discharged into the interior of the model, where its temperature was measured by the thermocouples indicated in Fig. 1. Finally, the helium escaped through the model porous wall into the adjacent boundary layer. Its temperature in leaving the model surface is assumed to be identical with the temperature of the outside model surface. This latter temperature was measured by iron–constantan thermocouples welded to the inside of the stainless steel shell, on the assumption that any temperature differences across the thin shell is negligible. The above pair of assumptions have been justified in [3].

The helium injection process, described above,

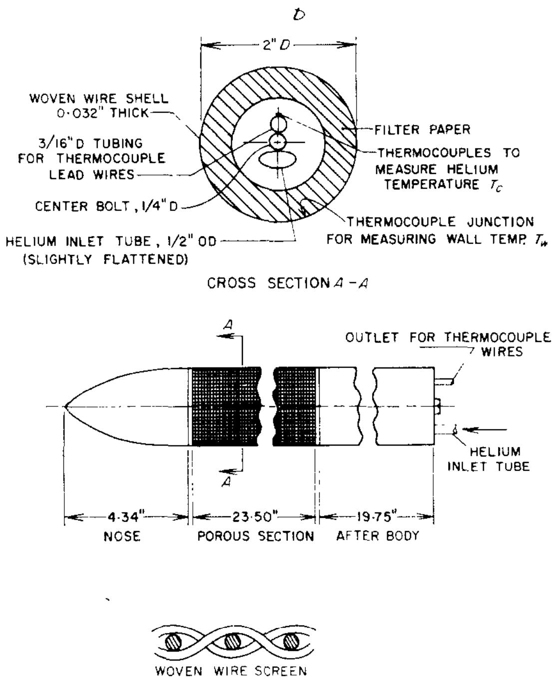

FIG. 1. Model and instrumentation.

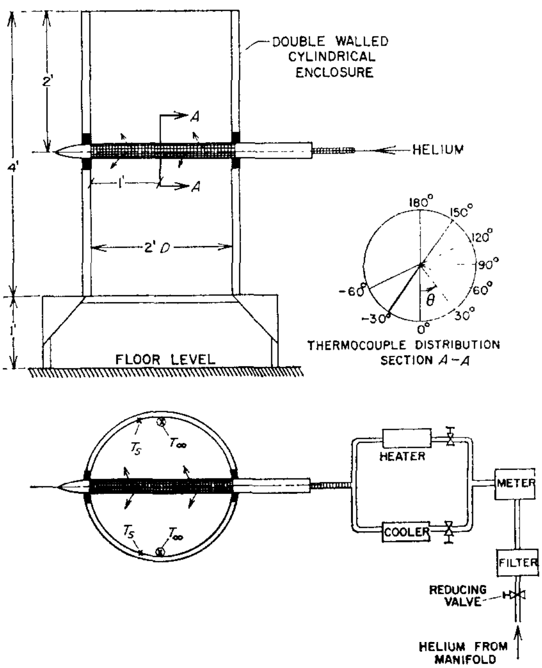

FIG. 2. Apparatus and helium flow system.

created concentration and temperature gradients across the boundary layers around the cylinder. Both gradients simultaneously contributed to the transfer of heat and to the transfer of mass throughout the boundary layer, according to the laws of thermodynamics of irreversible processes, e.g. [4].

The local heat flux exchanged between model and boundary layer by free convection is determined by applying the first law of thermodynamics to the open system shown in Fig. 3, with the result

$$q_w = \dot{m}C_p(T_c - T_w) - q_r - q_c \qquad (1)$$

in which q_r and q_c are small corrections due to radiation and circumferential heat conduction respectively, and are given by:

$$q_r = \epsilon\sigma(T_w^4 - T_s^4) \qquad (2)$$

$$q_c = -k_s b(\mathrm{d}^2T_w/\mathrm{d}\theta^2)/r^2. \qquad (3)$$

By measuring $\dot{m}$, T_c, T_w and T_s the local convective heat flux was computed from equation (1). The emissivity of the porous shell was determined by measurement to be 0·46.

3. EXPERIMENTAL PROCEDURE

The helium mass flow rate per unit area was fixed at a certain value, and the helium temperature inside the model was adjusted to several

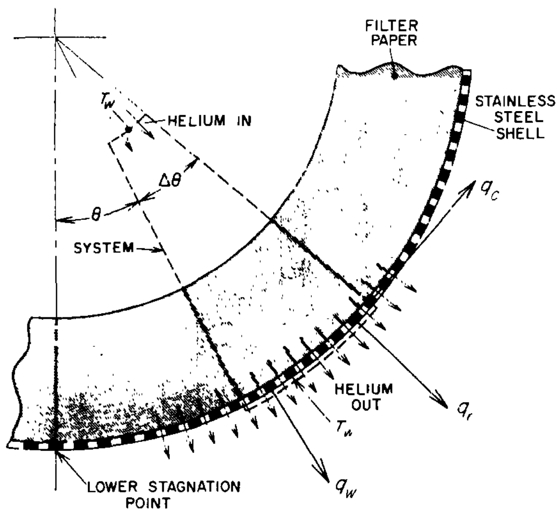

FIG. 3. Thermodynamic system to determine local heat flux.

levels, at each of which the wall temperature distribution was read, as well as T_∞ and T_s. In this way sufficient data were taken to determine the heat flux distribution around the cylinder at several levels of wall temperature, but at the same injection rate.

The above procedure was repeated for other values of the helium injection rate. In determining the heat flux, $\dot{m}$ was assumed to be uniform over the model surface. This assumption was determined to be correct within ± 4 per cent [3].

The ambient temperature T_∞ away from the model surface was measured by two shielded thermocouples located inside the enclosure. Their readings agreed within $\pm 0{\cdot}3$ degF, and their average was always used in the subsequent calculation. The temperature T_s of the inside wall of the enclosure was measured by two thermocouples taped to the wall. Their readings agreed within $\pm 0{\cdot}1$ degF.

All readings were taken, after steady state conditions had been satisfactorily established by observing successive readings of a particular thermocouple and verifying that they did not show a variation of more than 0·1 degF during 15 min intervals. Steady state was usually established after about 90 min for the smallest injection rate, and about 45 min for the highest injection rate. The data were taken at different days and reproducibility was checked occasionally.

4. RESULTS AND DISCUSSION

Measurements were taken at each of three to eleven different wall temperature levels for each of the six different helium injection rates indicated in Table 1 as cases 1 to 6. In each of the six cases, $\dot{m}$, T_∞ and T_s changed slightly from one run to another, and so the values indicated in the table are the mean of all the runs constituting one particular case. The variations from the mean in each case are indicated in the table.

The distribution of T_w around the circumference of the cylinder is shown in Fig. 4 for two runs in case 1, and one run in case 6. Similar distributions for all other runs were obtained.

A. *Heat transfer at the lower stagnation point*

From the measurements, the convective heat

Table 1

Case	Number of temperature levels	$\dot{m}$ ($lb_m/h\ ft^2$)	T_∞ (°F)	T_s (°F)
1	7	1·00 ± 1 per cent	65·9 ± 4	65·1 ± 4
2	3	1·49 ± 1 per cent	77·2 ± 3	78·3 ± 3
3	11	1·99 ± 1 per cent	71·7 ± 2	70·7 ± 2
4	8	3·99 ± 0·7 per cent	72·4 ± 4	71·9 ± 4
5	4	6·00 ± 0·5 per cent	76·2 ± 2	77·7 ± 2
6	4	7·50 ± 0·3 per cent	77·6 ± 3	79·5 ± 4

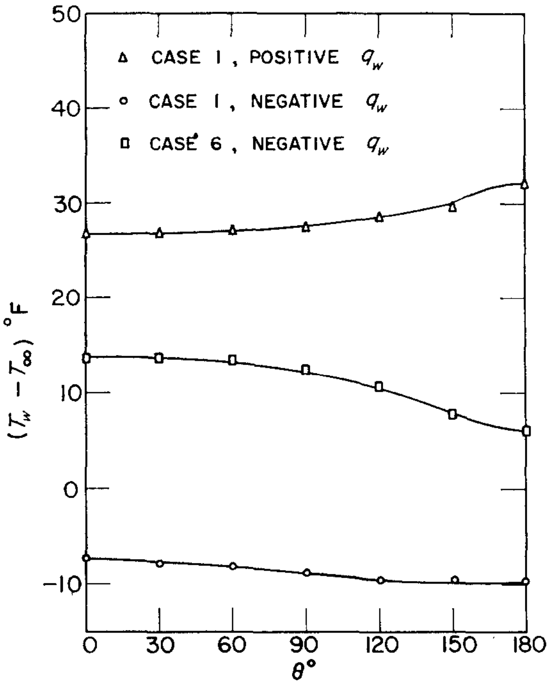

FIG. 4. Distribution of wall temperature.

flux at the lower stagnation point was computed with equation (1), and the results are presented in Fig. 5 for all cases 1 to 6. Two conclusions may be immediately made, concerning the relationship between q_w and T_w at a given injection rate:

(1) The heat flux is a linear function of the difference between wall temperature and ambient temperature.

(2) The heat flux does not vanish when the wall temperature is equal to the ambient temperature. On the contrary, q_w vanishes when T_w is higher than T_∞ by up to 89 degF depending upon the injection rate.

The first conclusion is quite different from other known cases of free convection, where q_w varies as some power of $(T_w - T_\infty)$ greater than unity, usually 5/4 or 4/3 power.

The second conclusion is also contrary to heretofore reported observations in free convection, in which q_w vanishes whenever $T_w = T_\infty$. However, it is similar to observations in forced convection, with helium injection into a low speed turbulent boundary layer [1, 2], where it was shown that the phenomenon must be due to thermo diffusion. Hence diffusion thermo-effects are also responsible for this apparent anomaly observed in free convection.

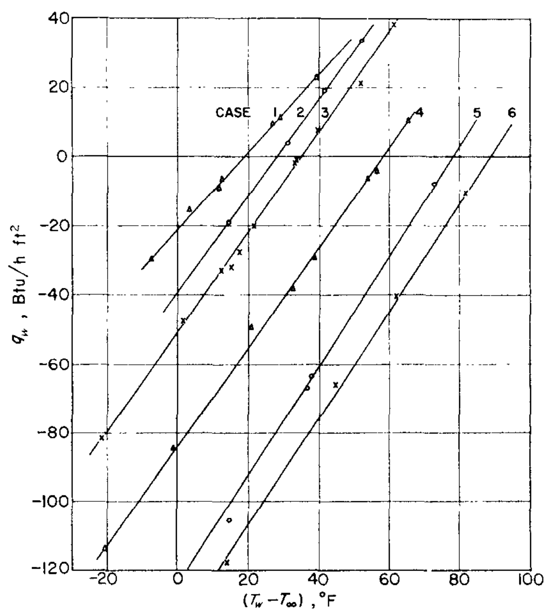

FIG. 5. Heat flux at lower stagnation point.

Denoting the wall temperature at which the convective heat flux vanished by T_a, the adiabatic wall temperature, it is possible to define a heat-transfer coefficient by

$$q_w = h(T_w - T_a). \quad (4)$$

Since q_w varies linearly with $(T_w - T_a)$ at a particular injection rate, h has a constant value independent of T_w, and equal to the slope of the various straight lines in Fig. 5. Again, the fact that h is independent of wall temperature level is contrary to the known cases of free convection in which h increases with T_w according to some power of $(T_w - T_\infty)$, usually 1/4 or 1/3.

From Fig. 5, the value of h at each injection rate was computed, and also $(T_a - T_\infty)$ determined. The results are presented in Fig. 6. As the helium injection rate increased, the heat-transfer coefficient increased rapidly at first, and then became almost constant for injection rates higher than about 2 $lb_m/h\ ft^2$. However the adiabatic wall temperature increased steadily with injection. Similar trends are evident by inspection of the dimensionless presentation of the dependence of the heat-transfer coefficient and the adiabatic wall temperature on the helium injection rate shown in Fig. 7.

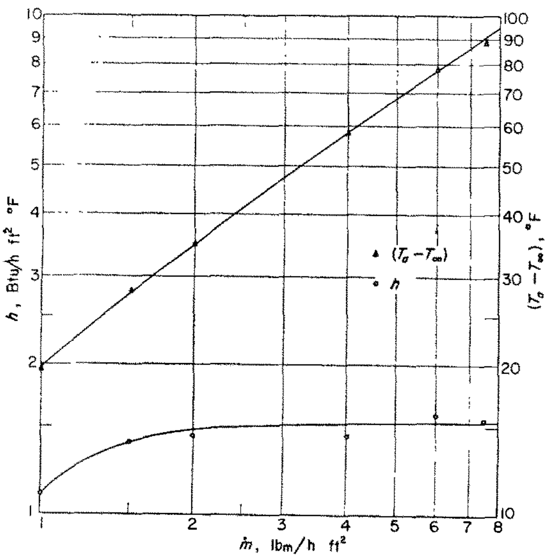

FIG. 6. Effect of injection rate on heat-transfer coefficient and adiabatic wall temperature at lower stagnation point.

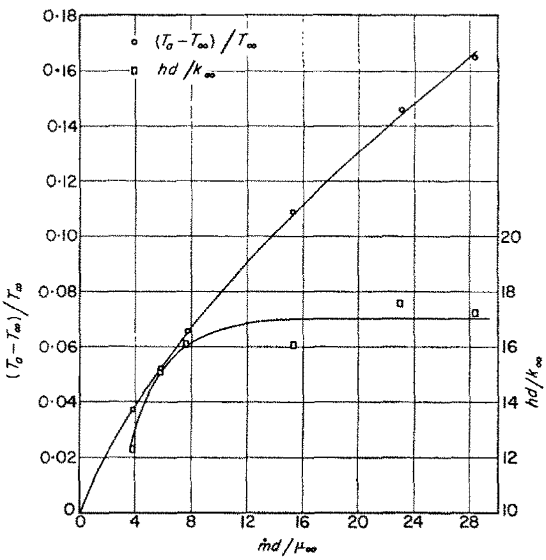

FIG. 7. Dimensionless presentation of heat-transfer coefficient and adiabatic wall temperature at lower stagnation point.

The injection of helium into a boundary layer tends to make the mixture with air lighter than the ambient air. This effect alone creates a buoyancy force which produces an upward motion in the boundary layer. Heating or cooling of the boundary layer will also contribute to the density variation in the boundary layer and in this way to the body forces. Hence two effects were at work in the present measurements, the effect due to the presence of a gas lighter than air such as helium in the boundary layer and the effect due to reduction of density by heating above ambient temperature. They aided each other when T_w was higher than T_∞, and opposed each other when T_w was less than T_∞. The latter effect appears to be negligible with respect to heat transfer as compared with the former, since the heat-transfer coefficient at a particular angle θ was shown to be insensitive to positive or negative values of $(T_w - T_\infty)$, even at injection rates as small as 1 $lb_m/h\ ft^2$; and since, on the other hand, it increased from 1 to 2 $lb_m/h\ ft^2$, due to increased buoyancy force when more helium was injected into the boundary layer.

It is known that the injection process tends to increase the thickness of the boundary layer and hence introduces more resistance to convective heat exchange, with a resulting drop in heat-transfer coefficient. This trend apparently worked opposite to the trend of improvement of heat-transfer coefficient due to increased buoyancy forces. Hence, beyond a certain injection rate, the two effects apparently balance each other and the heat-transfer coefficient will remain constant. This might explain the fact observable in Fig. 6 that h reaches constant value when $\dot{m}$ exceeded approximately 2 $lb_m/h\ ft^2$.

B. *Heat-transfer distribution*

By a procedure similar to that explained above for the lower stagnation point, the local heat flux was computed at all cases of injection, and at all locations around the cylinder. Typical results are presented in Fig. 8 for the upper stagnation point. Again the independency of the heat-transfer coefficient of wall temperature level, and the existence of an adiabatic wall temperature which is higher than the ambient temperature by up to 190 degF are evident. From figures such as Fig. 8 but constructed for the other circumferential locations, the distributions of the heat-transfer coefficient and of the adiabatic wall temperature around the cylinder circumference were determined. The results are presented in Figs. 9a and b, and 10. Although in some cases the cylinder wall was colder than ambient temperature by up to 20 degF, the heat-transfer coefficient at the lower stagnation point was always higher than that at the upper stagnation point. This is obviously a consequence of the fact that the buoyancy force due to helium injection, which has an upward direction, is the dominant one.

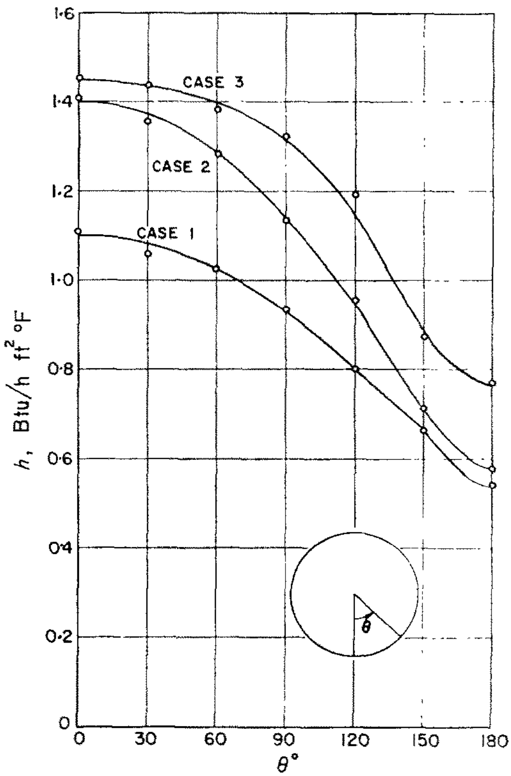

FIG. 9a. Distribution of heat-transfer coefficient.

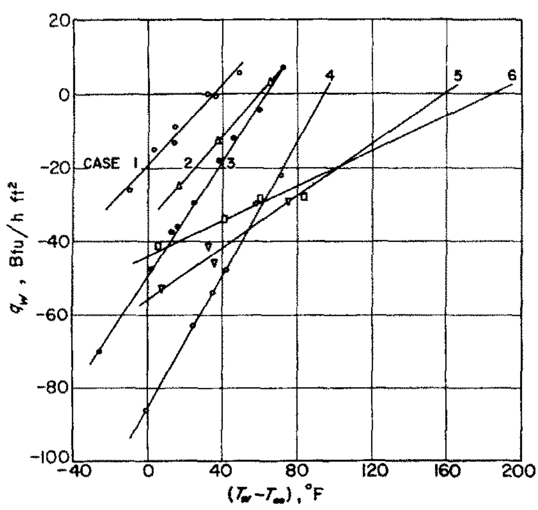

FIG. 8. Convective heat flux at upper stagnation point.

The heat-transfer coefficient decreased from a maximum value at the lower stagnation point to a minimum value at the upper stagnation point. The reduction is about 50 per cent for cases 1, 2, 3, and 4, but is much larger in cases 5 and 6, being about 80 per cent. In these two latter cases, the distribution resembles the familiar one around a solid wall cylinder in free convection.

The adiabatic wall temperature shown in Fig. 10, increases from a minimum value at the lower stagnation point to a maximum at the upper stagnation point. This increase is par-

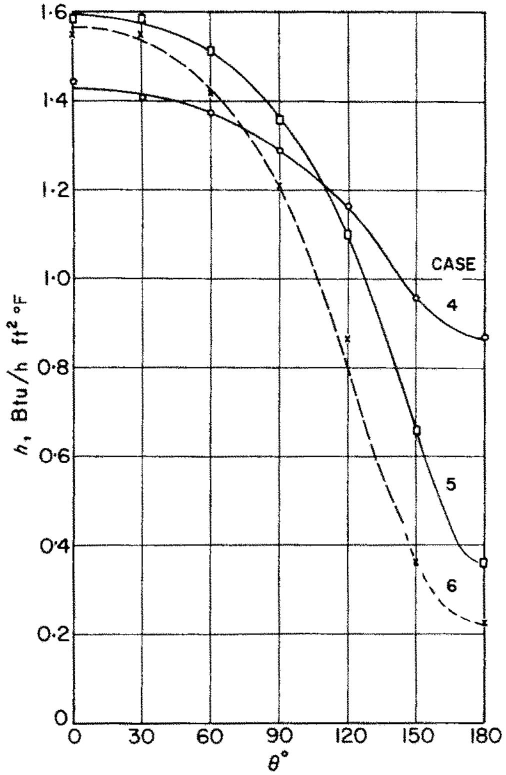

FIG. 9b. Distribution of heat-transfer coefficient.

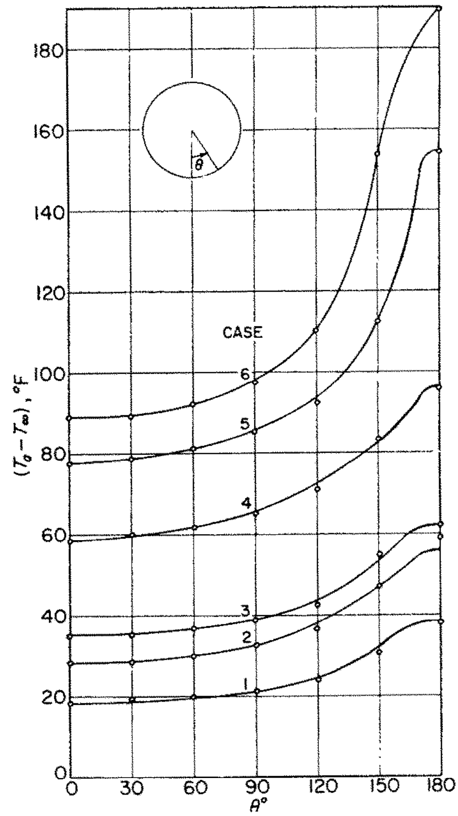

FIG. 10. Distribution of adiabatic wall temperature.

ticularly pronounced in cases 5 and 6, and its magnitude is similar to the drastic drop in heat-transfer coefficient from the lower to the upper stagnation points observed earlier for the same cases. At a particular location on the cylinder, the adiabatic wall temperature increased as the helium injection rate increased.

5. SUMMARY AND CONCLUSIONS

Local heat transfer by free convection was measured around a circular cylinder mounted with its axis horizontal in an isothermal enclosure. Helium was injected through the porous cylinder wall into the adjacent boundary layer, at six different mass injection rates, and at various levels of wall temperature at each injection rate. It was found that, when the convective heat flux defined by (1) vanished, the wall temperature, under this condition termed the adiabatic wall temperature, was higher than the ambient temperature by up to 190 degF, depending upon location around the cylinder and on helium injection rate. Similar phenomena were reported in other cases of helium injection [1, 2], and were shown to be due to diffusion thermo-effects.

In terms of the adiabatic wall temperature, a local heat-transfer coefficient was defined. Its value was found to be independent of the difference between wall temperature and ambient temperature, in the range of injection rates of these measurements.

Two effects contribute to the body forces acting on the fluid in the binary boundary layer during the experiments, the density changes caused by temperature differences in the boundary layer; and the reduction in density in the boundary layer with helium injection.

Since at a given injection rate, the local heat-transfer coefficient was found to be independent of the temperature difference between wall and ambient, and its value at the lower stagnation point was always higher than at the upper stagnation point whether heating or cooling occurred, the temperature differences must have a negligible effect on the heat transfer.

At the lower stagnation point, the heat-transfer coefficient increased rapidly with the helium injection rate, but then leveled off to a constant value when the injection rate exceeded about 2 $lb_m/h\ ft^2$. In this latter case, it seems that the tendency to improve the heat transfer by stronger buoyancy forces due to increased helium injection, just balances the tendency to decrease the heat transfer due to the thickening of the boundary layer as more helium was injected.

At a given injection rate, the heat-transfer coefficient was a maximum at the lower stagnation point, and decreased to a minimum at the upper stagnation point. In this respect, the distribution of the heat-transfer coefficient is similar to that in free convection from a solid wall cylinder at a temperature higher than ambient. The adiabatic wall temperature on the other hand increased from a minimum value at the lower stagnation point to a maximum value at the upper stagnation point.

Due to the large excess of adiabatic wall temperature over the ambient temperature found in the present measurements, heat fluxes to the wall can take place when its temperature is less than ambient temperature with a magnitude equal to or even greater than the heat flux in the case of zero injection. Hence helium injection seems to be of dubious value as a means of reducing heat transfer by free convection to a surface.

REFERENCES

1. O. E. Tewfik, E. R. G. Eckert and C. J. Shirtliffe, Thermal diffusion-effects on energy transfer in a turbulent boundary layer with helium injection. Proceedings of the 1962 Heat Transfer and Fluid Mechanics Institute, 42–61 (1962).
2. O. E. Tewfik and C. J. Shirtliffe, On the coupling between heat and mass transfer, *J. Aerospace Sci.* **29**, 8, 1009–1010 (1962).
3. O. E. Tewfik, E. R. G. Eckert and L. S. Jurewicz, Measurement of heat transfer from a circular cylinder to an axial air stream with air injection into a turbulent boundary layer. AFOSR 1397 (1961).
4. J. R. Baron, The binary-mixture boundary layer associated with mass transfer cooling at high speed. MIT Naval Supersonic Laboratory TR 160 (1956).

Résumé—On décrit ici des mesures de transmission de chaleur locale due à la convection libre en régime permanent autour d'un cylindre circulaire de 5 cm de diamètre, placé horizontalement dans une enceinte isotherme, pour plusieurs niveaux de température de paroi s'échelonnant de 11°C à 45°C au-dessus de la température ambiante.

A travers la paroi poreuse du cylindre on injecte de l'hélium dans la couche limite, à débit constant par unité de surface extérieure du cylindre (compris entre 5 et 38 kg/h.m.).

L'étude des résultats montre que lorsque le flux de chaleur convectif est nul, la température de paroi, appelée dans ce cas température de paroi adiabatique, est supérieure à la température ambiante d'une valeur pouvant atteindre 100°C, selon la position sur le cylindre et le taux d'injection d'hélium. Ce phénomène jamais relaté en convection libre est similaire à ceux des références [1, 2] et est dû ici à un effet thermique de diffusion.

Le coefficient local d'échange thermique est défini en fonction de la température de paroi adiabatique et on établit qu'il est essentiellement indépendant de la différence entre la température ambiante et la température de paroi. Au point d'arrêt inférieur, sa valeur croit rapidement avec l'injection, mais atteint presqu'une valeur constante quand le taux d'injection dépasse environ 10 kg/h.m². Pour un taux d'injection donné, le coefficient de transmission thermique décroit d'une valeur maximum au point d'arrêt inférieur à une valeur minimum au point d'arrêt supérieur.

En un point donné de la périphérie du cylindre, la température de paroi adiabatique s'élève régulièrement avec le taux d'injection. Pour un taux d'injection donné, la température de paroi adiabatique croît d'une valeur minimum au point d'arrêt inférieur à une valeur maximum au point d'arrêt supérieur. Les forces en présence dans la couche limite cont étudiées pour voir leur influence sur le transfert de chaleur.

Zusammenfassung—Es werden Messungen beschrieben für den stationären Wärmeübergang durch freie Konvektion an einem Kreiszylinder von 5 cm Aussendurchmesser, der horizontal in einem grossen Behälter liegt und Wandtemperaturen von etwa 11 grd unter bis etwa 44 grd über Umgebungstemperatur aufweisen kann. Durch die poröse Wand des Zylinders wurde Helium im konstanten Mengenstrom pro Einheit der Zylinderoberfläche im Bereich $1{,}36 \cdot 10^{-3}$–$10{,}2 \cdot 10^{-3}$ kg/m²s in die Grenzschicht eingeblasen.

Die Analyse der Ergebnisse zeigte für den Konvektionswärmestrom Null eine Erhöhung der Wandtemperatur, die in diesem Fall adiabate Wandtemperatur genannt wird, gegenüber der Umgebungstemperatur bis zu 105 grd, abhängig von der Lage am Zylinder und der eingeblasenen Heliummenge. Dieses, bisher bei freier Konvektion nicht angegebene Phänomen, ähnelt den Ergebnissen der Referenzen [1, 2] und beruht danach auf dem Diffusionsthermoeffekt.

Der örtliche Wärmeübergangskoeffizient wurde als Funktion der adiabaten Wandtemperatur definiert und im wesentlichen als unabhängig von der Temperaturdifferenz zwischen Wand und Umgebungermittelt. Am unteren Staupunkt nimmt sein Wert mit der Einblasung rasch zu, nähert sich aber dann einem beinahe konstanten Wert für Einblasmengen grösser als $2{,}72 \cdot 10^{-3}$ kg/m²s.

Bei gegebener Einblasmenge fällt der Wärmeübergangskoeffizient von einem Grösstwert am unteren Staupunkt auf einen Kleinstwert am oberen Staupunkt.

An ein und derselben Stelle am Zylinderumfang erhöht sich die adiabate Wandtemperatur stetig mit zunehmender Einblasmenge. Für gegebene Einblasmenge wächst die adiabate Wandtemperatur vom Kleinstwert am unteren auf den Grösstwert am oberen Staupunkt.

Die auf die binäre Grenzschicht wirkenden Kräfte werden hinsichtlich ihrer Bedeutung für den Wärmeübergang diskutiert.

Аннотация—Описаны измерения локального коэффициента теплообмена путём свободной конвекции в стационарных условиях для круглого цилиндра с наружным диаметром 2 дюйма. Цилиндр окружен изотермической оболочкой большого размера, а его ось занимает горизонтальное положение. Температура стенки изменялась от температуры приблизительно на 20°F ниже температуры окружающей среды до температуры приблизительно на 80°F выше температуры окружающей среды.

Гелий вдувался через пористую стенку цилиндра в пограничный слой с постоянной скоростью на единицу площади наружной поверхности цилиндра, диапазон скоростей: 1,00–7,50 фунт m/час кв.фут.

Анализ результатов показывает, что при конвективном тепловом потоке, равном нулю, температура стенки, называемая в этом случае адиабатической температурой стенки, была выше температуры окружающей среды приблизительно на 190°F и зависила от ориентации относительно цилиндра и скорости вдува ге́лия. Это явление, о котором до сих пор не сообщалось в работах по свободной конвекции, аналогично рассмотренному в [1,2] и, следовательно, обязано термодиффузии.

Локальный коэффициент теплообмена выражался через адиабатическую температуру стенки. Установлено, что он, в основном, не зависит от разности температур стенки и окружающей среды. В нижней критической точке значение коэффициента быстро возрастало с ростом вдува, но затем выравнивалось до почти постоянной величины при скорости вдува выше 2 фунтов m/час кв.фут. При данной скорости вдува значение коэффициента теплообмена уменьшалось от максимального в нижней критической точке до минимального в верхней критической точке.

В фиксированном месте цилиндрической поверхности адиабатическая температура стенки неуклонно возрастала с ростом скорости вдува. При данной скорости вдува адиабатическая температура стенки возрастала от минимального значения в нижней критической точке до максимального в верхней критической точке.

Рассматривалось влияние на перенос тепла массовых сил, действующих в бинарном пограничном слое.